Linear temperature dependence of the conductivity in Si two-dimensional electrons near the apparent metal-to-insulator transition

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In a high mobility two-dimensional electron system in Si, near the critical density, \( n_c = 0.32 \times 10^{11} \text{cm}^{-2} \), of the apparent metal-to-insulator transition, the conductivity displays a linear temperature \( T \) dependence around the Fermi temperature. When \( \sigma_0 \), the extrapolated \( T=0 \) conductivity from the linear \( T \) dependence, is plotted as a function of density, two regimes with different \( \sigma_0(n) \) relations are seen, suggestive of two different phases. Interestingly, a sharp transition between these two regimes coincides with \( n_c \), and \( \sigma_0 \) of the transition is \(-e^2/h\), the quantum conductance, per square. Toward \( T=0 \), the data deviate from linear \( \sigma(T) \) relation and we discuss the possible percolation type of transition in our Si sample.

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The nature of the ground state of an interacting two-dimensional electron system (2DES) in the presence of disorder is a long-standing problem in condensed matter physics. More than a decade ago, an apparent 2D metal-to-insulator transition (MIT) was first reported as the density of a Si metal-oxide-semiconductor field-effect transistor (MOSFET) is reduced through a characteristic density \( n_c \). Despite much research effort later in the field, there remain several unsettled fundamental questions. For example, metallic behavior is observed for density \( n > n_c \). Does this metallic-like state persist down to \( T = 0 \) and thus represent a true 2D metal? Furthermore, is this phenomenon related to a phase transition due to strong electron-electron (e-e) interactions, or is it a mundane crossover just due to a complex combination of many well-understood physical mechanisms?

Most of previous work was focused on the e-e interaction effect on metallic side of MIT. In recent years, however, it is becoming clear that in the regime close to the Wigner crystallization the strong e-e interaction also plays an important role in electronic transport behavior. In ultraclean 2D hole systems realized in the heterostructure insulating-gate field-effect transistors (HIGFETs), at the carrier densities around \( 10^9 \text{cm}^{-2} \), the 2D conductivity displayed a roughly linear \( T \) dependence near the Fermi energy. This linear dependence does not fit into any known single-particle picture and thus highlights the role of strong e-e interactions.

So far, almost all the experiments in this ultralow density limit have been exclusively carried out in the GaAs-based system, where the high sample quality allows one to approach an electron density down to \( 0.7 \times 10^9 \text{cm}^{-2} \). On the other hand, compared to many experiments already carried out in the conventional Si-MOSFETs, the studies in the ultralow density regime have been virtually zero, mainly due to their relatively poor sample quality in Si-MOSFETs, the highest mobility hovers around \( 4 \times 10^9 \text{cm}^2/V\text{s} \), while in GaAs HIGFETs the 2D hole mobility was reported to reach \( 1.8 \times 10^9 \text{cm}^2/V\text{s} \) (Ref. 7). With the advent of high-quality Si/SiGe heterostructures, it finally becomes possible to probe electron transport in low-density regime in the Si-based system, where the electron system is close to the regime of Wigner crystallization with strong e-e interactions.

In an earlier publication, we reported the observation of an apparent 2D MIT and the high-density metallic-like state and its response to an in-plane magnetic field in a high mobility Si quantum well. In this Brief Report, we focus on the transport properties for densities close to the transition. Here, the conductivity displays a linear temperature dependence near \( T=T_F \) (the Fermi temperature) and the slope is the same for all different densities around \( n_c \). When the extrapolated \( T=0 \) conductivity \( \sigma_0 \) of this transition is plotted as a function of density, two regimes with linear \( \sigma_0 \) vs \( n \) relations are readily seen. Interestingly, these two \( \sigma_0(n) \) lines cross each other exactly at \( n_c \) and \( \sigma_0 \) at the crossing coincides with the quantum conductance \( e^2/h \) per square. At low temperatures, \( T\ll T_F \), the measured \( \sigma(T) \) deviates from the linear \( T \) dependence. We discuss the low-\( T \) behavior of our data within the percolation model.

The experiments were performed on the 2DES in an n-type Si quantum well confined in a Si$_{0.75}$Ge$_{0.25}$/Si/Si$_{0.75}$Ge$_{0.25}$ heterostructure. The 2D electron density is tuned continuously by applying a front gate voltage to our field-effect transistor device. Details of the growth and the sample structure can be found in Ref. 10. Standard low-frequency (\( \sim 7 \text{Hz} \)) lock-in techniques were used to measure the 2D resistivity \( \rho \). At \( T \sim 300 \text{mK} \) and zero gate voltage, the 2DES has a density \( n = 1.45 \times 10^{11} \text{cm}^{-2} \) and mobility \( \mu = 190 \text{000 cm}^2/\text{V}\text{s} \).

In Fig. 1(a), we reproduce selected T-dependence data \( \rho(T) \) from our previous paper. Toward the \( T=0 \) limit, the apparent 2D MIT is clearly seen at the critical density \( n_c = 0.32 \times 10^{11} \text{cm}^{-2} \), where \( d\rho/dT \rightarrow 0 \) for \( T < 1 \text{K} \). The insulating behavior, \( d\rho/dT < 0 \), is observed for densities below \( n_c \) and metallic-like behavior, \( d\rho/dT > 0 \), above \( n_c \). For further insights to the 2D MIT, we now focus on the data around \( n_c \) and plot the inverse resistivity, or conductivity \( \sigma \), as a function of temperature in the density range of \( 0.27 \times 10^{11} < n < 0.38 \times 10^{11} \text{cm}^{-2} \) in Fig. 1(b). In the low-\( T \) limit, the MIT is again observed in that \( d\sigma/dT \) changes sign as \( n \) changes through \( n_c \). At high temperatures when \( T \) approaches the Fermi temperature \( T_F \), marked as short lines for each density, however, all \( \sigma(T) \) curves show roughly a linear \( T \) dependence and bends slightly downward for \( T \) sufficiently higher than \( T_F \).
samples. As a result, d compared to the much smaller number, temperatures in these two systems. As will be shown below, magnitude difference might be related to the different Fermi factor of 10 larger than that. Finally, we notice that a linear T dependence of the data, we 

temperature in the low-density regime. In contrast, d is nearly independent of the electron density around n. Under the temperature-dependent screening model, dσ/dT is proportional to 1/T or n−1. As a result, the slope should decrease by a factor of ~1.5 when the 2DES density increases from 0.27 to 0.38×10^{11} cm−2. On the other hand, the microemulsion model shows dσ/dT~n^2. Consequently, in the same density range, the slope should change roughly by a factor of 2. Finally, according to Ref. 5, the slope is proportional to τ, which is known to have a strong density dependence in the low-density regime. In contrast, dσ/dT is nearly constant and ~0.43±0.01e^2/h per Kelvin in our measurements. So far, it is not known what is responsible for this inconsistency between our experimental result and the theoretical predictions.

Having discussed the linear T dependence of the data, we
need to address some puzzling aspects of the overall results. First, the examined specimen has very high electron mobility and the \( n_c \) of the 2D MIT is by far the lowest among all the Si-based samples. The e-e interaction parameter \( r_s \) at the transition density is \( \sim 10 \), i.e., the Coulomb energy \( E_C \) exceeds the Fermi energy \( E_F \) by a factor of 20, after taking into account the twofold valley degeneracy in (001) Si 2DES. Consequently, even though the system behaves classically for \( T \sim T_F \), the 2DES is still strongly correlated since \( E_C \) is the dominant energy scale here. Second, if the high-\( T \) physics underlying the linear \( T/H_9251 \) vs \( T/H_11011 \) behavior persists to the \( T=0 \) limit, our \( \sigma_0(n) \) data show two distinct regimes, possibly suggesting two different electronic phases below and above \( n_c \). The transition between the two density regimes is sharp, and occurs almost exactly at \( n_c \). \( \sigma_0 \) at this transition point is very close to \( 2e^2/h \), the unit of quantum conductance, per square, manifesting a possible quantum nature of this transition. The reduction of the rate, \( d\sigma_0/dn \), of the density dependence, from \( \sim 26e^2/h \) per \( 10^{11} \) cm\(^{-2} \) above \( n_c \), to \( \sim 11e^2/h \) per \( 10^{11} \) cm\(^{-2} \) below \( n_c \), clearly indicates that in the low-density regime the 2D electrons are less likely to become localized by reducing the 2DES density. Considering the density regime the 2D electrons are less likely to become localized by reducing the 2DES density. Consequently, even though the system behaves classically at very low densities, a good linear fit with \( 1/r_s^2 \) is obtained over two decades in \( \sigma \). The slope corresponds to an exponent of \( \alpha=1.31 \).

We now turn to the low-\( T \) limit of the measured \( \sigma(T) \) in Fig. 1(b). Instead of following the trend from high temperatures, the conductivity deviates from the linear \( T \) dependence toward the \( T=0 \) ground state configuration. It has long been suggested that the apparent MIT observed here might be of a density inhomogeneity effect, and belongs to a general class of 2D percolation problem.\(^{16,17} \) In this picture, at very low densities, the 2DES is macroscopically inhomogeneous and first forms isolated puddles by occupying the low potential “valleys.” As \( n \) increases, the area of the electron puddles increases and at the percolation threshold density \( n_p \), some puddles are connected, giving rise to a conducting path throughout the sample. Experimentally, this percolation-type 2D MIT is supported by various measurements, e.g., scanning near-field optical microscopy,\(^{18} \) scanning single-electron transistor microscopy,\(^{19} \) transport,\(^{20} \) and surface-acoustic wave\(^{21} \) experiments.

According to the percolation model, the conductivity of the 2DES follows the scaling function \( \sigma \sim (n/n_p-1)^{1.13} \), and the exponent \( \alpha \) in a classical percolation transition is \( 4/3 \) (Ref. 14). In Fig. 3, we show the measured \( \sigma \) at \( T=0.3 \) K as a function of \( n/n_p-1 \) in a log-log scale. Since \( n_p \) is not known beforehand from finite temperature measurements, three trial densities of \( n_p \) are used, in units of \( 10^{11} \) cm\(^{-2} \), \( n_1=0.23 \) (extrapolation to \( \sigma_0=0 \) from the low-\( n \) regime), \( n_2=0.28 \) (extrapolation to \( \sigma_0=0 \) from the high-\( n \) regime), and \( n_3=0.32 \) (critical density of the observed 2D MIT). For \( n_p=n_2=0.28 \times 10^{11} \) cm\(^{-2} \), except for some deviation at very low densities, a good linear fit with \( \alpha=1.31 \), close to \( 4/3 \) in the classical model, can be obtained over two decades in \( \sigma \). Here, we want to point out that the deviation from the above percolation fitting occurs at the density where the “kink” in Fig. 2 is observed. This, again, suggests that the electronic phase in the low-density regime is different from that in the high-density regime. As for \( n_p=n_2 \), it is not known, at this stage, whether this coincidence is accidental or the two densities are actually deeply related. On the other hand, for \( n_p=n_1 \) or \( n_c \), the fittings to a power-law behavior are poor. We nevertheless emphasize two important points when applying the percolation model to our data. First, the power law of conductivity in the percolation model only holds for densities very close to \( n_p \), or \( n/n_p-1 \ll 1. \)\(^{13} \) Second, percolation transition is essentially a zero-temperature phase transition. As seen in Fig. 1(b), \( \sigma(T) \) does not saturate at our lowest measured \( T=0.3 \) K. In this regard, it is necessary that further measurements be carried out at lower temperatures.

In summary, in a high-quality Si quantum well specimen, near the apparent 2D metal-to-insulator transition, a linear temperature dependence of conductivity is observed at \( T \) around \( T_F \) on both sides of \( n_c \). When \( \sigma_0 \), the extrapolation of this linear \( \sigma(T) \) to \( T=0 \), is plotted as a function of density, two regimes with different \( \sigma_0 \) vs \( n \) relations are readily seen. Interestingly, the two linear \( \sigma_0(n) \) regions cross almost exactly at \( n_c \), and \( \sigma_0 \) at the crossing point is \( 2e^2/h \), the quantum conductance, per square. We also show that the measured \( \sigma(T) \) at our low-\( T \) limit can be fitted by a percolation scaling function \( \sigma \sim (n/n_p-1)^{1.13} \) when \( n_p=n_2 \), the extrapolation of \( \sigma_0(n) \) to \( T=0 \) on the high-density side.

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4 See, for example, S. Das Sarma and E. H. Hwang, Solid State Commun. 135, 579 (2005) and references therein.


8 J. Huang, D. S. Novikov, D. C. Tsui, L. N. Pfeiffer, and K. W. West, e-print cond-mat/0603184.


At low densities, the conductivity deviates slightly from the linear dependence. However, compared to the hopping fitting, where \( \sigma = \sigma_0 \exp\left[-(T_{\phi}/T)^{\alpha}\right] \) and \( \alpha = 1/3 \) or 1/2, the linear fit is still a better fit. Furthermore, according to Eq. (7) in Ref. 5, around \( T = T_F \), the conductivity is expected to be linear with \( T \).


13 We note that the viscosity of the electron liquid was also calculated by D. S. Novikov, e-print cond-mat/0603184, and is shown to be nonanalytical in the relevant temperature range.

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